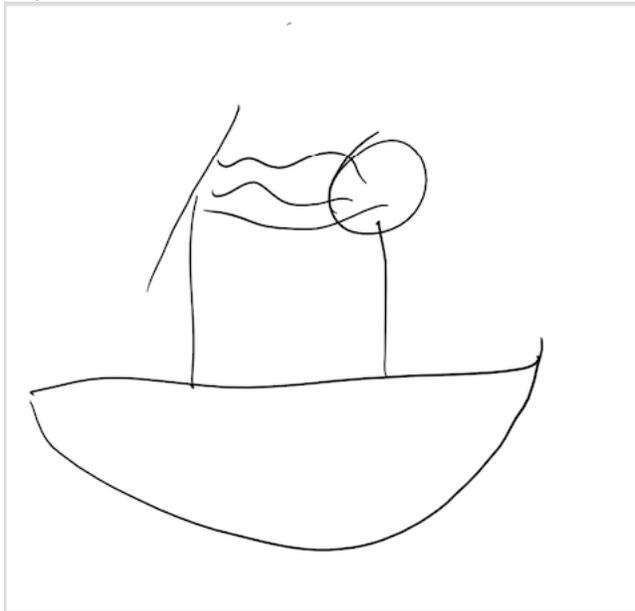# Physics Undergraduate Competition

April 12, 2017

1. You have a fan blowing wind a 2 m/s at a sail on a tilt θ relative to the vertical (assume the sail to be sturdy and perfectly flat). If the total mass of your ship is M, what is the speed of the boat after 10 minutes?

2. Explain in physical terms the design benefits of an Allen wrench (also known as Allen key or hex key).

3. Consider an infinitely nested Atwood machine (picture provided). What must the ratio of a mass on level x to a mass on a level n lower be in order for the infinite nested Atwood machines to be perfectly balanced?
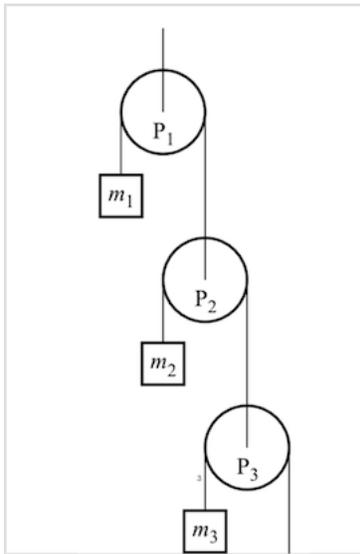
4. This is a classic problem about integration, and is borrowed from Harvard Professor Lyman's "*Problems of the Week*". Consider the curve $y = 1/x$, from $x = 1$ to $x = \infty$. Now rotate this curve around the x-axis to create a funnel-like surface. The volume of this funnel is

$$V = \int_1^\infty \frac{\pi}{x^2} dx = \pi$$

which is finite. The surface area, however, is:

$$A = \int_1^\infty \frac{2\pi\sqrt{1 - y'^2}}{x} dx > \int_1^\infty \frac{2\pi}{x} dx$$

which is *infinite*! So it seems like you can *fill up* the funnel with paint, but you *cannot* paint it! What is the solution to this apparent paradox?

5. Following are a series of something called *Fermi* questions. These questions are all about estimating the answer in powers of ten, so no need to *accurately* solve them. For every step of the calculation, round the numbers to the power of ten, and also leave the answer in powers of ten. If you are rounding a number, round down from 4 and round up from 5. Let us list the problems!

   (a) How many piano tuners are in Chicago?

   (b) How many red blood cells would it take to cover a tennis ball?

(c) How many minutes does it take for light to go from Earth to the Moon?

(d) 43^{35}

(e) $\frac{\text{Volume of a kilogram of feathers}}{\text{Volume of a kilogram of Lead}}$